DOCUMENT RESUME

ED 252 948 EA 017 458

TITLE High School Mathematics. Minimum Standards Implementation Series.
INSTITUTION Ohio State Dept. of Education, Columbus. Div. of Elementary and Secondary Education.
PUB DATE 83
NOTE 66p.; For related documents, see EA 017 445-459.
PUB TYPE Legal/Legislative/Regulatory Materials (090) -- Guides - Non-Classroom Use (055)

EDRS PRICE MF01/PC03 Plus Postage.
DESCRIPTORS Competency Based Education; High Schools; *Mathematics Education; Mathematics Skills; *Minimum Competencies; *Program Development; *Program Evaluation; Secondary Education; *Secondary School Mathematics
IDENTIFIERS *Ohio; Ohio State Board of Education

ABSTRACT

One of a series of implementation documents prepared in conjunction with the revised minimum standards adopted in 1983 by the Ohio State Board of Education, this publication is designed to assist those charged with program development and review in reviewing their high school programs to determine whether their mathematics curricula meet the revised standards. Part I examines the general mathematics offerings at the secondary level, part II discusses goals of the programs, and part III identifies alternatives for meeting the new second-unit requirements for graduation. (DCS)

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HIGH SCHOOL MATHEMATICS

MINIMUM STANDARDS IMPLEMENTATION SERIES

OHIO DEPARTMENT OF EDUCATION
COLUMBUS, OHIO
1983

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FOREWORD

With the adoption of the 1983 minimum standards, the State Board of Education has established conditions to promote a general education of high quality in all chartered schools. The State Board of Education recognizes its responsibility to give direction to the system of education in the state. At the same time, there is flexibility within standards to establish educational programs that are responsive to local needs.

Minimum Standards for Elementary and Secondary Schools 1983 provide the potential for greatly improving education in Ohio. That potential can be reached through well-structured local and state leadership. To assist responsible implementation of minimum standards, staff in the Ohio Department of Education have prepared an implementation series consisting of fourteen documents. A distinction is made among the publications. Only one, *Procedures for School Evaluations,* is a compliance document. It contains the standards in a format readily used for self-evaluation. School personnel will find information about what is needed for a district and schools within the district to comply with standards. In the remaining publications, an effort is made to specify what the standard requires. Additional information, guidelines, procedural steps, and sample forms may go beyond what the standard requires. The purpose is to assist the local planning process, not to prescribe what shall be in every school in Ohio.

Minimum standards address the public's increasing expectations for schools with a requirement for competency based education in English composition, mathematics, and reading. Three of the publications, *Process Model for Course of Study, Competency Assessment and Testing,* and *Intervention* are aligned in addressing competency based education. A number of options are described for local consideration.

Appreciation is extended to school personnel who contributed to these publications by sharing their time, expertise, and materials. Appreciation is also extended to staff members who worked untold hours to make these publications possible.

Franklin B. Walter

Franklin B. Walter
Superintendent of Public Instruction

TABLE OF CONTENTS

Preface 3

Introduction 4

PART I

General Mathematics Defined 5
Status of General Mathematics 6
Minimum Standards 7
Bridging the Gap 7

PART II

New General Mathematics Program 9
A New Ninth Grade General Mathematics Course 11
Entering Skills 11
Course Description 13
Representative Ninth Grade New General Mathematics Skill Areas 15
Representative Textbook Pages 18

PART III

Options for Meeting the Second Unit of Mathematics Required for Graduation 22
Informal Statistics 25
New General Mathematics II 30
Consumer Mathematics 32
Consumer Simulation Mathematics 33
Consumer Applied Mathematics 33
Problem Solving Mathematics 38
Vocational Mathematics 41
Business Mathematics 42
Vocational Orientation Mathematics 43
Vocational Cluster Mathematics 45
Microcomputer Applications to Mathematics 51
Competency Mathematics 53

References 58

PREFACE

Providing the most appropriate mathematics curriculum for all students is a challenging responsibility. Mathematics has become an indispensable tool. More than at any time in our past, knowledge and appreciation of mathematics have become essential to participate fully in society. Young people with a sound preparation in mathematics will have more career choices.

The newly adopted Minimum Standards for Ohio Elementary and Secondary Schools have increased the mathematics requirement for high school graduation from one unit to two. These standards also focus upon improvement of learner outcomes and emphasize meeting pupil needs in a high technology society. The impetus, therefore, exists for school district personnel to review their high school programs and to determine whether their mathematics curricula meet today's challenge. It is hoped this document will assist those charged with program review and development.

INTRODUCTION

In December 1982, the State Board of Education adopted Minimum Standards for Ohio Elementary and Secondary Schools. Among the new requirements are two units of mathematics for high school graduation. The increased mathematics requirement acknowledges the expanding importance of mathematics in personal, vocational, and social responsibilities.

The new standards also require districts to adopt competency based curricula with periodic assessments of pupil performance and testing at least once in grades one through four, five through eight, and nine through eleven. Suitable intervention procedures are expected for pupils not meeting the competency requirements. The emphasis in the minimum standards is upon improving curricula and assuring quality of educational performance. With these purposes in mind, this document consists of three parts. Part I examines the general mathematics offerings at the secondary level. Part II discusses goals of the program. Part III identifies alternatives for meeting the second unit requirement for graduation.

*DID YOU KNOW?

- 68 percent of Ohio students were enrolled in two or more years of mathematics;
- In 81 percent of Ohio schools, more than half of all students were enrolled in two years of mathematics;
- General mathematics was the fourth most frequently scheduled course (89 percent of the schools scheduled general mathematics, which followed Algebra I, Geometry, and Algebra II in frequency);
- A question requesting the title of the general mathematics course(s) taught produced 51 different titles, the largest list on any similar question about secondary courses;
- 61 percent of Ohio secondary schools (9-12) added one or more mathematics courses in the last three years;
- The frequency of new course listings by topic were computers, calculus, consumer mathematics, probability and statistics, and prealgebra.

*These statistics are from a stratified random sample of Ohio secondary schools conducted during the spring of 1981.

DEFINITION

PART I

GENERAL MATHEMATICS DEFINED

The general mathematics program consists of those courses for grades 9-12 which are not considered requirements for college preparation. Students taking coursework in general mathematics will not necessarily terminate their education with high school. However, the content, approaches, and applications of this coursework have a more personal, vocational, and utilitarian nature than the college preparatory program.

The mathematics experiences making up general mathematics alternatives are expected to fulfill four functions:

1. to provide the mathematics preparation necessary for active citizenship in a democratic society;
2. to extend the individual's capability and confidence to apply mathematics for personal needs;
3. to supply the general background of academic skills and knowledge required for entry and subsequent training in a chosen vocation;
4. to increase the individual's appreciation of the importance of mathematics in communicating, learning, and thinking.

These functions identify the key elements of mathematics education for an individual. Mathematics is both a tool and a means to communicate. The vast array of quantitative data and information confronted daily makes mathematics a fundamental tool for exercising responsible citizenship. It is an essential means for becoming a contributing member of society. It provides the individual with a necessary capability for satisfying personal needs and interests. It is a language and structure for communicating ideas to others and to one's self.

The following student goals identify the rich and diverse applications of mathematics that each individual will need as an adult:*

a) to recognize and use the mathematics necessary for one's own consumer affairs, selecting and using goods and services wisely—choosing and buying personal items, products for a home, for a family, or for a small business;
b) to recognize and use the mathematics necessary for other quantitative aspects of one's daily life—mathematics related to cooking, sewing, gardening, minor home or auto repair, personal projects, money management, and recreation;

*Adopted with permission from *Applications of General Mathematics: A Prospectus for the Development of Video Materials for Secondary School Students*, (Agency for Instructional Television, Bloomington, Indiana, 1979).

c) to possess requisite mathematics to enter a job situation—appropriate background, learning skills, and attitudes so that additional job-related applications of mathematics can be learned with relative ease;
d) to use logical reasoning and problem solving skills in reaching decisions, weighing evidence, using data, and interpreting information to arrive at sound decisions and conclusions.

Achievement of these student goals requires understanding and reasonable competence in arithmetic computation including oral, paper-pencil, and calculator components. This understanding and competence must be supported by:

- estimation and approximation skills;
- understanding of, and ability to use, simple algebraic formulas;
- ability to interpret and analyze graphs and statistical information;
- knowledge of geometric forms and spatial relations;
- understanding of and ability to use measuring instruments including the relation of geometric and numerical concepts as the mathematics of measuring instruments;
- problem solving and logical thinking abilities, ranging from skill with simple word problems to more complicated analyses of situations requiring an awareness of productive reasoning techniques; and
- an awareness and appreciation of the capabilities and limitations of devices for carrying out mathematical transactions such as the calculator, computer, electronic banking machines, or food market price readers.

STATUS OF GENERAL MATHEMATICS

In the past, general mathematics programs have been only partially successful in achieving the student goals delineated in the previous section. Reasons for lack of success vary. It is clear, however, that the general mathematics program has not realized its potential as a parallel and desirable alternative to college preparatory coursework. Unlike the college preparatory program, most general mathematics programs do not possess a common, articulated body of content.

General mathematics frequently consists of remedial courses for less able, less successful, and less motivated students. There is a stigma attached to taking general mathematics courses and to teaching such courses. Because of the wide range of student abilities in many general mathematics courses, instruction sometimes only achieves the lowest common denominator. Computational skill development may be emphasized to the exclusion of activities involving the application of mathematics to real situations.

MINIMUM STANDARDS

As a consequence of this approach to general mathematics, many students perceive little relationship between classroom activities and the type of skills necessary in the real world. For too many students, mathematics never becomes an operational tool.

MINIMUM STANDARDS

One purpose of the minimum standards and mandated competency based education programs—with periodic assessments of student performance and intervention for those students not meeting expectations—is to assure that students develop the desired competencies as they progress toward graduation. The need for extensive remediation of elementary school topics at the secondary level should be minimized.

The opportunity now exists to strengthen general mathematics programs and to offer strong, alternative instruction for students who lack the interest or need (but not ability) for college preparatory coursework. The second unit of mathematics required for graduation is a strong incentive to develop new course sequences at the high school. If carefully articulated with seventh and eighth grade courses, these courses should help assure a well integrated and strong mathematical preparation—broad in nature through grade nine and more specialized thereafter.

Remedial coursework should not be considered part of the general mathematics curriculum. If remedial instruction is required as part of the district's intervention efforts, this instruction should be confined to a single course at the ninth grade level. The number of students needing such assistance should be limited. And the extent of remediation necessary to overcome deficiencies should be manageable within this one course. If this is not the case, it will indicate a failure of the competency program and intervention efforts at earlier grades.

Objectives, general approaches, and methodology for the broad preparation intended for a new ninth grade general mathematics course are described in subsequent sections. Some suitable alternatives for the second unit required for graduation are discussed.

BRIDGING THE GAP

The new general mathematics program offers a parallel and coequal alternative to the college preparatory program. This program is predicated on the assumption that students enrolled in a competency based mathematics program through the elementary grades will have significantly different mathematical needs and abilities.

BRIDGING THE GAP

The competency based program is not required to be fully implemented until the 1989-90 school year. The second unit of mathematics for graduation is not required until 1987. The effects of the competency based program at ninth grade may not be fully reflected for several years. Therefore, there may be an interim when the curriculum described for the new general mathematics program may not be appropriate for all students.

The new program will take time and planning to implement. For these reasons, careful study and phase-in are recommended. School personnel might:

- Review existing mathematics offerings for the noncollege preparatory student;
- Determine if appropriate courses exist for these students to meet the two-unit mathematics graduation requirement.

If appropriate course offerings do exist, school personnel can begin a study of the steps necessary to implement the new general mathematics program described in this document. If appropriate course offerings do not exist, school personnel can make short-term curricular adjustments that permit students to meet the two-unit requirement until skill levels are sufficient to implement the new general mathematics program.

Although an interim second general mathematics course may be remedial, instruction should be guided by the principles and guidelines prescribed for the new ninth grade general mathematics course. Teachers should emphasize the application of mathematical concepts and skills—not rote learning of concepts isolated from real world application. Calculators should be used extensively. Little or no time should be spent practicing paper and pencil computational algorithms.

In determining content for the interim course, a logical place to begin is the competencies established by the local competency program. Additional guidance can be found in the *suggested entry skills* and ninth grade *abilities* listed in this document. Course descriptions listed in the *options* section also may provide helpful suggestions.

The transition time, when the competency based education program is being fully implemented, may pose a challenge to districts not scheduling two appropriate units for noncollege preparatory students. Course development and implementation should proceed according to this tenet: as competency program begins to insure student competence. course goals, instructional methods, and expected outcomes will be adjusted accordingly.

NEW PROGRAM

PART II

NEW GENERAL MATHEMATICS PROGRAM

The following assumptions constitute a philosophical framework for the program:

- Learning to solve problems is the principal reason for studying mathematics. The mathematics curriculum should be organized around problem solving, as recommended by the National Council of Teachers of Mathematics in *The Agenda in Action**.
- Mathematical literacy for all students is the fundamental goal of the mathematics program. Literacy implies the student can apply mathematical knowledge and processes to satisfy routine personal, citizenship, and vocational needs.*
- Mathematics requirements in school districts in Ohio and other states are being increased, as supported by such professional organizations as the National Council of Teachers of Mathematics. Increasing such requirements makes it even more important to assure a comprehensive, articulated program of appropriate mathematics for the noncollege preparatory student.
- Students will benefit from at least one mathematics course that involves them realistically in the process of applying mathematics to nontrivial situations. This experience should assist students to apply whatever mathematics skills and knowledge they already possess rather than upon developing new concepts.
- The precalculus sequence (algebra, geometry, college algebra, elementary functions) provides students with the most options for further education and professional careers. However, all students of sufficient ability should not automatically be scheduled into this sequence. A combination of courses from the new general mathematics and college preparatory program could provide the best alternative for some students.
- The college preparatory and new general mathematics programs are not mutally exclusive. There are some courses for which students from each of these programs should naturally matriculate and perform satisfactorily. After suitable ninth or tenth grade prerequisites in either program, students should be prepared to take such courses as computer applications to mathematics, statistics and probability, and consumer mathematics.

NEW PROGRAM

- There should be only one remedial option in the high school program. This option should permit some students the opportunity to acquire missing skills. All other courses should be nonremedial in both content and approach. In particular, the second unit of mathematics required for graduation should not be an extension of a remedial program. Deficiencies of this magnitude should be identified and remedial prior to students entering the ninth grade.
- Pencil and paper computation is not prerequisite to the mathematics that many adults do, nor is pencil and paper computation more important than computation by estimation or by calculating devices.*
- The new general mathematics program should contain some significant content from each of the traditional subcategories of mathematics: arithmetic, algebra, geometry, trigonometry, statistics, probability.
- The new general mathematics curriculum should be sufficiently flexible to permit different forms of implementation in individual districts to meet the diverse needs of communities and students.*
- The success of the new general mathematics program depends largely upon attitudes. The acceptance of this program by teachers, the support from administrators, and the confidence and receptivity of students are critical in implementation. Given appropriate materials, curriculum, instruction, motivation, and follow-through, students can and will learn mathematics.

*Passages with an asterisk have been adopted from Chambers (1983, pp 203-204).

A NEW COURSE

A NEW NINTH GRADE GENERAL MATHEMATICS COURSE

Pupils in the new ninth grade general mathematics course will have successfully completed a competency based mathematics program through grade eight. The new course will draw upon the knowledge and skills resulting from this program.

ENTERING SKILLS

The following skills are neither a list of *minimum competencies* nor a statement of all the important mathematics taught in grades one through eight. Rather, the skills provide guidance for placing of students into a secondary mathematics course which is both non-college preparatory and nonremedial. The new ninth grade mathematics course would be inappropriate for students without a majority of these requisite skills.

Numbers and Numeration. The student is able to:

- recognize the place value of digits in numbers through billions.
- write a number in fractional, percent, or decimal notation given a rational number in any of the other forms.
- express a number given in exponential form in standard form.
- factor whole numbers into primes and find the greatest common factor and least common multiple of two numbers.
- round numbers to the nearest thousand, hundred, ten, one, tenth, hundredth.
- communicate using rational and irrational numbers.
- make meaningful interpretations of large numbers (e.g., conceptualize the national debt by using a personal referent).
- decide and use a hand-held calculator to do calculations with whole numbers and decimals when a calculator is more appropriate than pencil and paper or mental calculation.

Operations and Properties. The student is able to:

- perform the four basic operations on whole numbers:
  a. addition (four-digit numbers)
  b. subtraction (four-digit numbers)
  c. multiplication (three-digit numbers)
  d. division (two-digit divisor, four-digit dividend).
- use a standard algorithm for each of the four basic operations on reasonable (or common) fractions, decimals, mixed numbers, and integers.
- find the square root of a whole number less than 1000 to the nearest whole number.
- compare and order fractions, decimals, whole numbers, and integers.

ENTERING SKILLS

- apply mathematical properties in meaningful ways (e.g., commutative, associative, distributive, order of operations, division by zero to shorten calculations, recognize equivalent expressions).
- solve a proportion and use proportions to solve percent problems.

Geometry. The student is able to:

- recognize, sketch, and use the defining characteristics of horizontal lines, vertical lines, skew lines, parallel lines, perpendicular lines, and intersecting lines.
- classify and use the defining characteristics of simple plane and solid figures.
- compute perimeters and areas of common polygons and circles.
- apply the Pythagorean property.
- locate points on the coordinate plane (including positions on a map).
- name and sketch prisms, pyramids, cones, and spheres and identify defining characteristics.

Measurement. The student is able to:

- select and use appropriate units to measure length, area, volume, angles, weight, capacity, time, temperature, and money.
- convert from one common unit of measure to another within the same measurement system.
- read the scale on a measurement device to the nearest mark and make interpolations where appropriate.
- use the common metric prefixes.

Problem Solving. The student is able to:

- write a mathematical expression for a word phrase.
- draw upon a variety of strategies in solving mathematical problems (e.g., make an appropriate sketch, make a table, look for a pattern).
- solve traditional and nontraditional one or two-step problems presented in a worded format.
- draw logical conclusions, summarize accurately, and relate ideas and concepts found in different resource materials such as standard reference texts and indexes.
- identify irrelevant information in solving a problem and determine when given information is insufficient for a solution.
- solve first degree equations in one variable (e.g., $\square + 5 = 8, 3t = 9$).
- use graphs, charts, or tables to obtain needed information, identify patterns, note trends, and draw conclusions.

COURSE DESCRIPTION

Probability and Statistics. The student is able to:

- read, interpret, and construct tables, charts, and graphs using data drawn from real life.
- use elementary notions of probability to determine the likelihood of an event.
- find the mean, median, mode, and range for a set of data

Estimation and Approximation. The student is able to:

- execute approximate calculations by rounding numbers.
- use simple techniques for estimating quantity, length, distance, and weight.
- determine when a particular result is precise enough for a specified purpose.
- perform appropriate mental computations with two-digit whole numbers.

Attitudes. The student is able to:

- give examples of how frequently mathematical skills are used by individuals in their daily lives.
- work in a consistent and persistent manner when solving mathematical problems and exercises.
- exhibit confidence in being able to apply personal skills and knowledge to attack and solve problems.
- work independently on problems or exercises for a reasonable time before seeking assistance.

COURSE DESCRIPTION

Prior to the ninth grade, mathematics objectives primarily address knowledge and process outcomes. Ordinarily, less emphasis is put on application and problem solving outcomes. For example, a concept such as ratio (part-to-part comparisons) will have been introduced as an idea, practiced as a manipulative skill, and perhaps considered in isolated and specific application situations. Although exposed to applications, this exposure typically is limited to one- or two-step word problems which require *only* that concept of ratio for solution. The major reason for doing word problems is to practice specific skills in a nonexercise format. These problems usually do not require students to use a full repertoire of strategies, skills, and critical thinking abilities.

The new ninth grade general mathematics course will require students to integrate several concepts within the solution of a single problem. More than two steps will generally be required in reaching solutions. Problems will frequently be in word formats, contain extraneous information, or lack information that must be supplied by

the solver. Problems will not be restricted artificially to a minimum of words for the sole purpose of reducing the amount of reading required.

Prior to grade nine, learning objectives are generally approached in a developmental sequence of small and distinct steps. At grade nine, the same content may be explored again from a different perspective—the relationship of widely diverse applications of the same idea (e.g., ratio used for unit pricing, mileage, rates, large number comparisons). The purpose is to consolidate disparate views of the same idea into a unified concept or principle which may be more powerfully applied.

Problems should not be contrived or overly simple; they should strive to resemble real-life situations. Every effort should be made to apply knowledge and skills in contexts meaningful to ninth grade students. The process of bringing together various components required for problem solving should be emphasized rather than definitions or simple interpretations of given data or information. When calculations are to be made, the activity should reflect a real-life method—employing mental computation, estimation, approximation, and the use of calculator or computer, as well as traditional paper and pencil methods. Computation should be a means to an end, rather than an end in itself.

Applications should frequently require that students generate or compile information as part of the problem process. Real world materials and devices should be incorporated into coursework. Heavier emphasis should be placed upon laboratory types of instruction, group work, and longer range learning tasks and projects.

The capabilities below are representative of the kinds of skills students should acquire through the ninth grade new general mathematics course. Such skills should be developed in the context of realistic and substantive problem experiences. The list is not intended to be exhaustive, or stated in measurable terms. Expressed in broad, general statements, the list is *descriptive* of the kinds of activities and outcomes to be addressed in this course.

SKILL AREAS

REPRESENTATIVE NINTH GRADE NEW GENERAL MATHEMATICS SKILL AREAS

Using Numbers Executing Calculations. The student will be able to:

- make mental computations (add and subtract two-digit numbers mentally, use number properties to make calculations easier, round in +, −, ×, ÷ to minimize errors in carrying out approximations).
- select an appropriate computing method or device to suit the calculation task.
- ascertain the reasonableness of a computer result or given piece of information derived from numbers.
- use a four function calculator including K, $\sqrt{}$, x^2, and memory/storage keys.
- conceptualize or use referents to make very large or very small numbers personally meaningful.
- use a computer keyboard and describe the general purpose, use, and limitations of computers.
- make estimations using techniques such as rounding, compensation, proportions, partial count, referents, sampling.

Measurement/Geometry. The student will be able to:

- convert within the system between reasonable measures of the metric and customary systems of measurement.
- carry out length measurements in the appropriate system to fractions of an inch or to millimeters.
- determine measurements from the gauges or scales of measuring devices.
- apply perimeter, area, and volume determinations (in such things as fencing, paint coverage, air conditioner capabilities, cooking).
- apply geometric notions of parallel, perpendicular, subdivision into equal parts, similarity (carpentry, gardening, construction, home maintenance).
- use ideas and devices to make sketches or drawings (compasses, rulers, protractors, scales, vectors, graph paper, slopes).

Tables/Charts/Graphs. The student will be able to:

- read and interpret charts, tables, forms (mileage, postage, transportation, sales tax, income tax, sports, employment, banking).
- read and interpret graphs (bar, pictograph, circle, broken line, combinations of these).

SKILL AREAS

- read, interpret, and follow directions associated with schematic drawings (floor plans, maps, mechanical drawings, assembly instructions).

Ratio/Proportion/Percent/Scaling. The student will be able to:

- apply percent ideas (finding amounts, identifying percent parts of the total, determining changes in quantity, calculating discounts/markups, understanding the dependence of percent upon an established base).
- carry out cost determinations based on simple formula relationships (explicitly or implicitly given); e.g., the rental cost of a truck ($30 a day plus 20¢/mile) driven 982.5 miles in four days.
- make drawings using scales, protractor, ruler, and compass.
- apply the concepts of ratio and proportion (scale drawings, recipes, money management, and purchases).

Statistics. The student will be able to:

- read meaningfully materials that contain statistics (mean, median, mode, range, *average*, money amounts, variation of data).
- collect, aggregate, organize, present, and analyze data to study a situation requiring sampling of a population.

Formulas. The student will be able to:

- generate information for use in other parts of a problem.
- apply exponential ideas to compounding and simple growth situations involving formulas or tables.
- apply simple right triangle relationships through formulas and measurement (Pythagorean, sine, cosine, tangent relations).

Consumer Applications. The student will be able to:

- determine money amounts in transactions (making change, totaling bills, calculating tips, using cash registers).
- calculate costs (making unit cost determination for comparison; total cost for lists of materials; scaled costs for recipes, formulas, or budgets; financing and credit costs compared to cash costs).

Probability. The student will be able to:

- apply elementary notions of probability (listing the outcomes of an event, expressing probability as the ratio of successes to total outcomes, recognizing probability as a predictor for a large number of trials in applications such as weather forecasting, principles behind insurance, games of chance, sporting odds).

SKILL AREAS

Decision Making/Reasoning/Problem Solving. The student will be able to:

- apply mathematical reasoning to (a) produce counterexamples; (b) detect and describe flaws in arguments, advertising, propaganda that are dependent upon numerical data; (c) gather and present data to support an inference or argument.
- appreciate the strengths, limitations, and applications of deductive and inductive approaches to problem situations.
- display identifiable problem solving behaviors when confronting new situations to be resolved (sketching a diagram, solving a simpler problem first, making a table, working backwards).

REPRESENTATIVE TEXTBOOK PAGES

The following four pages show the positive movement some of the more progressive publishers have made over the traditional drill and practice approaches of the past.

REPRESENTATIVE TEXTBOOK PAGES

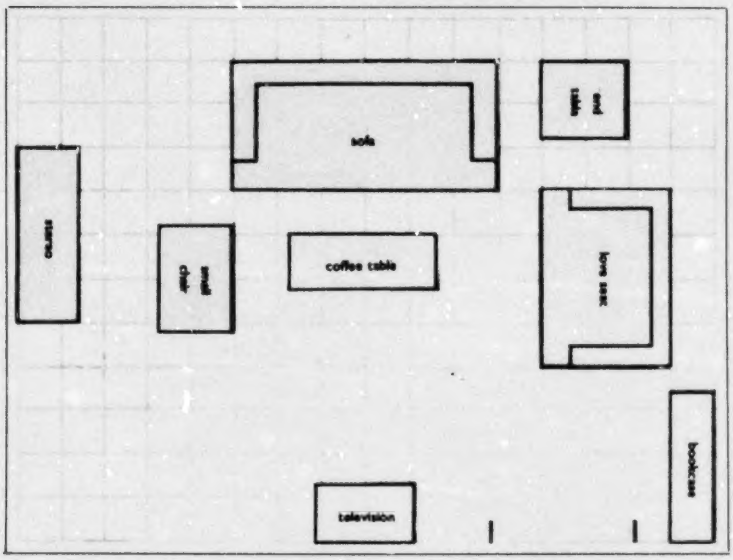
Objective: To learn how ratios are used in scale drawings.

9-7 SCALE DRAWINGS

Mike and Anita want to see how their furniture fits into the new house. They draw a model of the living room and models of their furniture.

Scale drawings are used to picture things which are too large to be drawn actual size. A scale drawing of the living room of the model home is shown below.

On this scale drawing, one inch represents 48 inches (4 feet).



On the scale drawing the sofa measures 1½ in. What is the length of the actual sofa? Use a proportion to find the actual length, l.

$$\frac{1}{48} = \frac{1\frac{1}{2}}{l}$$

$$1 \times l = 48 \times 1\tfrac{1}{2}$$

$$l = 72$$

The actual length of the sofa is 72 inches.

key skills

pp. 40, 41

$$4 \times \frac{3}{16} = \frac{4}{1} \times \frac{3}{16}$$

$$= \frac{4 \times 3}{1 \times 16}$$

$$= \frac{3}{4}$$

Multiply. Write each product in simplest form.

1. $2 \times \frac{5}{8} = 1\frac{1}{4}$
2. $4 \times \frac{3}{8} = 1\frac{1}{2}$
3. $7 \times \frac{3}{4} = 5\frac{1}{4}$
4. $48 \times \frac{5}{16} = 15$
5. $48 \times \frac{3}{4} = 36$
6. $48 \times \frac{1}{32} = 1\frac{1}{2}$
7. $1\frac{1}{2} \times \frac{3}{16} = \frac{9}{32}$
8. $4 \times \frac{11}{32} = 1\frac{3}{8}$
9. $48 \times 3\frac{5}{16} = 159$

REPRESENTATIVE TEXTBOOK PAGES

MAX: 1-10; AVG: 1-10; MIN: 1-8

Find the actual measurement of each of the following. The scale drawing measurement is given. Use the scale 1 inch represents 48 inches.

1. love seat, 1 in. 48 in.
2. width of the room, 3 in. 144 in.
3. length of the room, 4 in. 192 in.
4. bookcase, $\frac{3}{4}$ in. 36 in.
5. coffee table, $\frac{7}{8}$ in. 42 in.
6. television, $\frac{9}{16}$ in. 27 in.
7. end table, $\frac{7}{16}$ in. 21 in
8. small chair, $\frac{5}{8}$ in. 30 in.
9. width of the doorway, $\frac{13}{16}$ in. 39 in.
10. width of the coffee table, $\frac{11}{32}$ in. 16.5 in.

CAREER

AERIAL PHOTOGRAPHER

Jim Gibbons is an aerial photographer. He uses a camera to take photographs from the air. These photographs are used to make maps which agencies use to study water pollution, soil erosion, and so on. The photographs are a scale model of what is actually on land. Suppose 1 centimeter in the photograph is the same as 240 meters on land. The distance between two points on the photograph is 0.25 centimeters. What is the actual measurement?

$$\frac{1}{240} = \frac{0.25}{l}$$

$$l = 240 \times 0.25$$

$$l = 60$$

The actual length is 60 meters.

Find the actual length of each of the following. The length in the aerial photograph is given.

1. 3.5 cm 840 cm.
2. 0.7 cm 168 cm.
3. 5.2 cm 1,248 cm.
4. 0.85 cm 204 cm.

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REPRESENTATIVE TEXTBOOK PAGES

The following two pages characterize the project, process emphasis described for the new general mathematics ninth grade course.

PROJECT: DESIGNING A HOUSE

Materials: Drawing paper, 4 pieces of 1 cm graph paper, a centimetre ruler

PLANS

- Here's a chance to design your own house. Imagine you are 20 years older? Are you single or married? Do you have any children?
- Look at different houses in your community. Decide which ones you like best.

THINGS TO DO

1. Make sketches of the house, some of the rooms, and the landscape.
2. Draw a floor plan for the house.
3. Draw a floor plan of your favorite room. Put your own furniture designs in it.
4. Make a scale drawing of the lot the house is on.

SKETCHES

- Before you make up your mind about your house answer these questions.

  Would you like a house from your favorite period in history?
  Do you enjoy climbing stairs?
  What can be used in place of bricks or wood to build a house?
  Do you like wide-open spaces or closed-in places?
  Do you want to use solar energy?
- Sketch the front of your house. Below the sketch write the length of the house.
- Make a list of the rooms you'll need. Sketch two of them. Write down their lengths and widths.
- Sketch the lot. Write down the lengths of the sides of the lot.

REPRESENTATIVE TEXTBOOK PAGES

FLOOR PLANS

Now you're ready to design the inside of your house.

- Choose a good scale and draw a floor plan.
- Add doors and windows. Do all your rooms have windows? Should they?
- Health experts say a house should have 20 m^2 to 40 m^2 of floor space for each person. How much floor space do you have? Do you have enough floor space for each person?
- Every car needs about 16 m^2 of parking space. If you have a garage is it large enough?

ROOMS AND FURNITURE

Use another sheet of graph paper to draw your favorite room in the new house.

- Choose a scale. Draw a floor plan. Make a list of the furniture needed.
- Find good lengths and widths for the furniture. Make at least one piece of furniture to fit you.
- On another piece of graph paper draw the furniture. Use a room scale.
- Cut out the furniture. Arrange it on the room plan. How much usable floor space is there?
- Choose one of the carpeting styles listed at the bottom of page 84. How much would it cost to carpet your room?

LANDSCAPING

Use your last sheet of graph paper to draw the lot.

- Choose a scale. Draw the lot. Add the house.
- Use symbols to show the other things in your lot.
- Do you have a fence or wall around your lot? Even if you don't, find out how long a fence would be needed.

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OPTIONS

PART III

OPTIONS FOR MEETING THE SECOND UNIT OF MATHEMATICS REQUIRED FOR GRADUATION

The new general mathematics course should provide a sound foundation for students. This foundation will provide a firm basis upon which to build understandings of more advanced mathematical concepts and skills. Concepts and skills may be developed through one of several alternatives for the required second unit of mathematics. These options for the second unit of mathematics are based upon the five assumptions:

- Implementation of a mandated competency based education program in mathematics will help assure that fewer students arrive at the ninth grade with mathematical deficiencies.
- The existing need for a nonremedial course at the ninth grade level for noncollege preparatory students will be exacerbated. The broad framework for such a course has been outlined.
- Students who complete the competency based education program and the new ninth grade general mathematics course will possess a very different set of competencies, skills, knowledge, and attitudes than is now the case.
- Existing school course offerings for this student may no longer be adequate or appropriate, particularly in meeting the second unit required for graduation.
- The full implementation of a comprehensive new general mathematics curriculum may evolve over a period of time. Examination of the mathematics curricula for the noncollege preparatory student and the development of transitional activities should begin immediately.

OPTIONS

One alternative for students successfully completing the new ninth grade general mathematics is Algebra I. This course should be seriously considered. The National Council of Mathematics has recommended that "Algebra should be included in the program of all capable students to keep their options open." Students should be informed that a precalculus sequence provides them with the most latitude for further education and professional careers. But a precalculus sequence is not necessarily the best choice for all students.

Other alternatives must also be available. The following pages provide sample course descriptions of options designed for the students with the new general mathematics preparation. Although some of these courses have traditional names, the content or approach is usually not traditional. All options except competency mathematics are nonremedial. The list is not intended to be exhaustive or to suggest that a district implement all of the options. Rather, the list illlustrates the types of courses that may meet the needs of a noncollege preparatory student. Each option includes examples of objectives, examples of activities that teachers could use with students, and example problems. A summary of the options is given in Table 1.

TABLE I.

OPTIONS FOR THE SECOND UNIT OF MATHEMATICS*

COURSE OPTIONS	CONTENT	METHODS OF INSTRUCTION	POTENTIAL POPULATION
1. Informal Statistics	Application of statistical ideas, methods and formulas to real world situations. Some probability is incorporated.	Discovery, laboratory, experiential course. Minimal lecture. Computers and calculators desirable.	For any student completing New General Mathematics I or Algebra I.
2. New General Mathematics II	An extension and deepening of concepts and ideas from the new ninth grade course.	Hands-on approach. Heavy use of reference materials. Computers and calculators desirable.	Specifically for students in the noncollege preparatory program.
3. Consumer Simulation Mathematics	Applications of mathematics to adult life. Includes earning, spending, and saving money.	Simulation, use of real forms, charts, and data. Not text oriented. Computers and calculators desirable.	For any 11th or 12th grade student.
4. Applied Consumer Mathematics	Applications of mathematics to adult life. Includes earning, spending, and saving money.	Real world orientation. Some lecture, more use of text and word problems. Computers and calculators desirable.	For any 11th or 12th grade student.
5. Problem Solving Mathematics	Specific methods and techniques for solving mathematical problems.	Emphasis upon discovery and processes of thinking. Problems may be theoretical. Computers and calculators desirable.	For any student completing New General Mathematics I or Algebra I.
6. Business Mathematics	Application of algebra and arithmetic to common business situations.	Real world orientation. Lecture, use of text, and "real" forms. Computers and calculators desirable.	For students intending a business, but not a college, career.
7. Vocational Orientation Mathematics	Application of arithmetic, algebraic, geometric, and trigonometric skills to vocational situations.	Application, orientation, use of formulas and rules. Computers and calculators desirable.	For students intending to pursue an unspecified high school vocational program.
8. Vocational Cluster Mathematics	Application and reinforcement of basic, consumer, estimation, measurement, geometry, algebra, and trigonometry skills.	Laboratory approach. No isolated drill and practice. Computers and calculators desirable.	For students intending to pursue a specified high school vocational program.
9. Microcomputer Applications to Mathematics	Mathematical topics explored through the use of a microcomputer.	Intensive individual use of microcomputers as tools for solving mathematical problems. Computers are required.	For any student completing New General Mathematics I or Algebra I.
10. Competency Mathematics	Content determined by the district competency program and individual student needs.	Diagnostic, prescriptive teaching, individualization. Use of manipulatives. May require use of alternative algorithms and instructional methods. Computers and calculators desirable.	For students not meeting the district competency requirement.

*These options are intended to provide examples of possible courses. There are many other appropriate courses. Districts may want to develop their own courses, drawing upon suggestions from this document and elsewhere, or adapt an existing course from a source such as the National Diffusion Network.

INFORMAL STATISTICS

INFORMAL STATISTICS

Statistics is an important topic in an information age. The ability to comprehend numerical information has become a basic skill. Anyone wishing to understand the graphs, surveys, and other numerical information presented in magazines and newspapers needs this skill. Nearly all forms of decision making rely on numerical information. The course suggested here does not emphasize theoretical statistics or the memorization and application of statistical formulas. Rather, it consists of a hands-on, experiential course. The emphasis is on:

- "locating and processing quantitative information;
- collecting data;
- organizing and presenting data;
- interpreting data;
- drawing inferences and predicting from data." (National Council of Teachers of Mathematics, 1980, p. 7).

Students should use real data, and these data should relate to student concerns (favorite sport, favorite car, etc.). "Only by being involved in the handling of data from beginning to end will students lose their mistrust of things numerical and perhaps even gain an appreciation for the statistical way of thinking." (Noether, 1980, p. 196).

There are many sources for content outlines and suggested activities for this course. The following description should be considered one alternative. It is important that instructional strategies emphasize hands-on experience, projects, and experiments. Concepts and ideas are best developed in realistic application situations. Electronic aids such as computers and calculators should be used whenever possible and appropriate. This should not be a lecture/recitation course.

INFORMAL STATISTICS

EXAMPLE OBJECTIVES (Note: this is one of many possible lists of objectives appropriate for a high school course in informal statistics). The student will be able to:

- collect data from a variety of sources using a variety of methods (survey, observation, library research).
- group data into classes on the basis of a variety of attributes.
- determine frequency distributions from a set of raw data.
- sketch histograms.
- sketch frequency polygons and frequency curves.
- determine the relative and cumulative frequencies of a set of raw data.
- conduct experiments involving counting and measuring.
- determine measures of central tendency for raw data (range, median, mode, mean, standard deviation).
- determine percentiles and quantities of a set of raw data.
- determine a variety of measures of spread of dispersion about the mean (mean absolute deviation, standard deviation for raw data, standard deviation for group data).
- list and distinguish between several common statistical distributions.
- apply characteristics of the normal distribution.
- compare the distribution of a set of student-collected data with that of the normal distribution.
- compare relative frequencies in an experiment with theoretical probabilities.
- apply probability in situations involving life and automobile insurance.
- predict the performance of an athlete or team from past experiences.
- use statistical methods, formulas and facts in real-life problem solving and decision-making situations.

INFORMAL STATISTICS

EXAMPLE ACTIVITIES

1. Have students determine the make of automobile found most frequently on the school parking lot. Have students determine whether subcompact cars, compact cars, or full size cars are more popular. Have the students check the makes, sizes, or colors of cars found on the school parking lot during their mathematics class. If students and teachers park in different areas, ask the class to draw conclusions about the differences in the cars driven by the two groups. Have students check new car registrations to determine the current frequency of registration for the different types of vehicles.
2. Fill a clear glass jar with dried beans. Ask students to estimate the number of beans in the jar. Record the estimates and display them on a histogram. Does the histogram approximate the normal curve? Have students find the mean, median, mode, and standard deviation of the estimates. Have the students determine which student most closely estimated the number of beans and whether the class as a whole did well. Try the experiment again with a larger jar. Did students' estimating ability improve?
3. During a semester or year long activity, students can develop a price index analogous to the consumer price index. Have students take a survey to determine where they spend their money and which commodities are most frequently purchased. Students should collect and evaluate the surveys. A *basket* of goods and services representing things most important and most frequently purchased is identified. Pricing and indexing procedures should be explained, and students should carry them out. Have students price selected items each month in selected stores on given days. (Lenoski, 1981)

EXAMPLE PROBLEMS

These problems are illustrative of type, difficulty, and level of interest.

1. The following list gives the heights of girls on the high school track team:
   60″, 62″, 63″, 60″, 61″, 61″, 61″, 62″, 60″, 64″, 65″, 66″, 68″, 61″, 63″, 62″, 62″, 65″, 66″, 64″, 60″, 58″, 63″, 60″, 63″.
   a. Make a frequency table.
   b. Find the mean, median, and mode.
   c. Does the mean alone give an accurate description?
   d. Does stating the median and mode along with the mean provide a more accurate impression of the data?
   Explain.

INFORMAL STATISTICS

2. In 1875, Walter Slade of Great Britain ran a mile in 4 minutes, 24.5 seconds. This was the fastest anyone had ever been timed. Now high school students often do better. Here is how the record has evolved in the years since 1875.

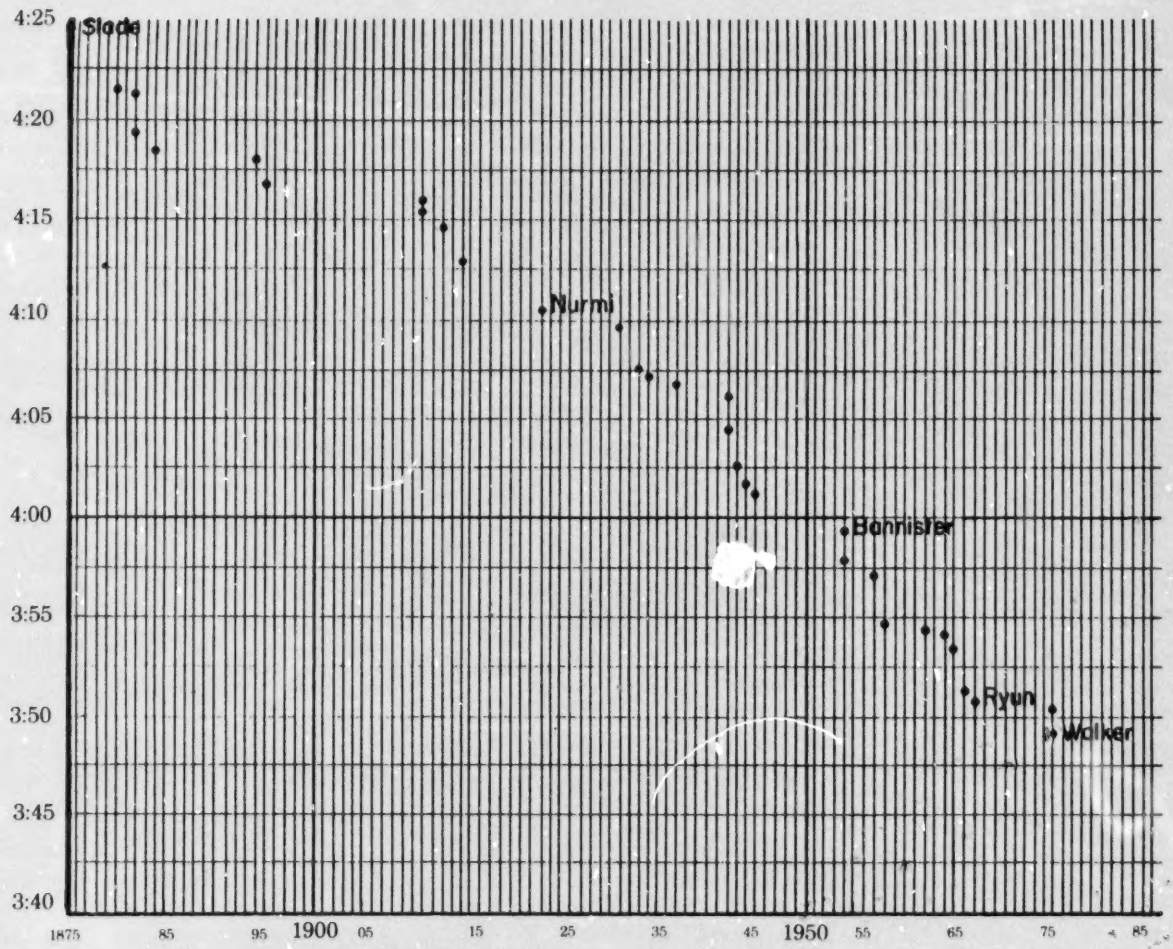
1875	Walter Slade, Britain	4:24.5
1880	Walter George, Britain	4:23.2
1882	George	4:21.4
1882	George	4:19.4
1884	George	4:18.4
1894	Fred Bacon, Scotland	4:18.2
1895	Bacon	4:17
1911	Thomas Connett, U.S.	4:15.6
1911	John Paul Jones, U.S.	4:15.4
1913	Jones	4:14.6
1915	Norman Taber, U.S.	4:12.6
1923	Paavo Nurmi, Finland	4:10.4
1931	Jules Ladoumegue, France	4:09.2
1933	Jack Lovelock, New Zealand	4:07.6
1934	Glenn Cunningham, U.S.	4:06.8
1937	Sydney Wooderson, Britain	4:06.4
1942	Gunder Haegg, Sweden	4:06.2
1942	Arne Andersson, Sweden	4:06.2
1942	Haegg	4:04.6
1943	Andersson	4:02.6
1944	Andersson	4:01.6
1945	Haegg	4:01.4
1954	Roger Bannister, Britain	3:59.4
1954	John Landry, Australia	3:58
1957	Derek Ibbotson, Britain	3:57.2
1958	Herb Elliott, Australia	3:54.5
1962	Peter Snell, New Zealand	3:54.4
1964	Snell	3:54.1
1965	Michel Jazy, France	3:53.6
1966	Jim Ryun, U.S.	3:51.3
1967	Ryun	3:51.1
1975	Filbert Bayi, Tanzania	3:51.0
1975	John Walker, New Zealand	3:49.4

The ordered pairs (year and record) are graphed on the next page. Since 1900 the points lie very close to the line through 1900, (4:20) and 1975, (3:50). The record has not yet bottomed out.

INFORMAL STATISTICS

a. Give an equation for this line using year and seconds.
b. Since 1900, what is the largest amount the record has deviated from the time predicted by the equation in (a)?
c. What does the slope of the equation in (a) signify?
d. Use the equation to predict what the record will be in 1990.
e. Show the limitations of fitting this data to a line by predicting what the record will be in the year 2980.

Evolution of the Mile Record 1875-1975*



*From Mathematical Association of America and the National Council of Teachers of Mathematics (1980, pp 128-129). Subsequent citations to this publication will be abbreviated MAA and NCTM.

NEW GENERAL MATHEMATICS II

NEW GENERAL MATHEMATICS II

This course focuses on the mathematics necessary to function effectively and efficiently in a technological, democratic society. It is an extension of New General Mathematics I and should be considered the second course in a two-course sequence.

The goal of this course is to make students functionally competent to apply mathematics to daily citizenship tasks and personal activities and problems. These tasks, activities and problems include traditional *consumer topics* (i.e., comparative shopping, budgeting, checking accounts) appropriate for tenth grade students, but emphasize more generic mathematical skills such as problem analysis and interpretation.

Instructional experiences are intended to help students successfully apply classroom concepts to real-life tasks. This can be accomplished by examining the unifying ideas in seemingly diverse applications and by exploring in detail how these ideas are related and applied. While this is an obvious continuation of activities begun in New General Mathematics I, the second course would include a greater emphasis on projects.

EXAMPLE OBJECTIVES (Note: the content of this course involves applications of a greater variety and depth than for New General Mathematics I; however, content topics are the same. This course extends and reinforces content from the previous course. For a more detailed discussion of objectives, see pages 15 through 17.)

The student will be able to:

- apply computational skills, which include selection of appropriate computing method or device, facility in mental computation, and judgment of the reasonableness of results.
- use a computer in nonprogramming applications.
- make and use measurements.
- apply geometric ideas in interpreting and working in the three-dimensional world.
- read, interpret, and construct tables, charts, and graphs.
- apply ratio, proportion, and percent ideas in every day situations.
- collect, organize, and analyze data and make appropriate decisions on the basis of the analysis.
- use formulas in solving numeric and geometric problems.
- use mathematics to make decisions involving money.
- apply elementary notions of probability.
- apply mathematical reasoning and specific problem solving strategies.

NEW GENERAL MATHEMATICS II

EXAMPLE ACTIVITIES

1. Make arrangements with a local realtor to have students visit a vacant house. Have students take measurements and make scale drawings of the house and grounds surrounding the house.
2. Using a catalog or pamphlets from local furniture stores, have students select furniture for the house. Have students construct floor plans to show where the furniture will be located. Assign students a given amount for the furniture purchase. Have them calculate the total cost of their selections, including tax, and arrange a financing plan for its payment.

EXAMPLE PROBLEMS

New General Mathematics II should provide opportunities to deepen and extend the concepts and ideas from the ninth grade course. For example, a possible extension, related to the designing a house project, is the study of residential energy use. The project could require students to do research to obtain data, or information could be included in the problems.

"In recent years, about 18 percent of all the energy consumed in the United States has been used in homes" (U.S. Department of Energy, 1978, p. 1). The breakdown of energy use within homes indicates that "Fifty-four percent of 1980 residential energy consumption was used for heating homes and 19 percent for heating water" (U.S. Department of Energy, 1982, p. 33). The remaining "Twenty-seven percent of 1980 residential energy consumption was used for nonheating uses such as air conditioning, cooking, lighting, refrigeration, washing, drying, and various other primary electricity using services" (U.S. Department of Energy, 1982, p. 33).

1. In an all-electric home in Ohio, the electric bill for January was $193.62 and the bill for June was $87.21.
   a. If 54% of each bill is for home heating, how much was spent each in January and June for heating?
   b. Is it reasonable to assume that 54% of each bill is for home heating? Explain.
   c. If 19% of each bill is for heating water, how much was spent in January and in June for heating water?
   d. Is it reasonable to assume that 19% of each bill is for heating water? Explain.
2. If electricity costs six cents per kilowatt hour, how many kilowatt hours were used in January and in June?
3. During January the thermostat at the Smiths was set on 68°F. The heat cost can be reduced by 2.3% for every degree the thermostat is lowered. If the day time (6:00 A.M. to 10:00 P.M.) temperature setting is lowered to 66°F and the night time setting is lowered to 63°F, by how much should the heating bill be reduced?

4. The Smiths' 17-year-old son, Butch, watches television 12 hours a day. If the wattage on the television is 150, this means it uses 150 watts of electricity each hour. Butch's father would like to know how much it costs to have the television set on 12 hours a day. He says it is probably several dollars daily. Butch maintains it is only a few cents. If a kilowatt is 1000 watts and a kilowatt hour costs six cents, who is right, Butch or his father?
5. Mrs. Jones has told Mr. Jones that they simply *must* cut their monthly electric bill by at least $15.00. The Jones know the following approximate monthly costs:

Freezer (20 cu. ft.)	$12.04
Dishwasher	1.80
Clothes washer	4.20
Water bed heater	8.10
Color television	1.80
Yard light	3.60
Clothes dryer	1.44
Tub baths (Mrs. Jones)	4.50
Showers (Mr. Jones)	3.60

   a. What are some of the different options the Jones have for cutting their electric bill?
   b. Which option is best? Why?

CONSUMER MATHEMATICS (11th-12th grade)

This course is appropriate for more mature students: it deals with consumer applications of mathematics in adult life. The course is equally valuable for college preparatory and noncollege preparatory students. There are two alternatives for consumer mathematics provided. The objectives of both courses are the same. The difference arises in the methodology and type of student activities.

Flexibility is desirable in developing a consumer mathematics course. Topics needed by students in different localities should be adjusted to reflect local job opportunities or differing backgrounds. The two consumer courses described here represent only two of many possibilities. A more complete description of the first course may be found in *Applications in School Mathematics*. The second suggested course is a more typical consumer mathematics course, but it emphasizes real world experience rather than pencil and paper work in the classroom. While example objectives and problems are the same for both courses, the activities used to reach the objectives are different.

CONSUMER MATHEMATICS

CONSUMER SIMULATION MATHEMATICS (a semester or year long simulation of typical problems of everyday living).

This is a simulation designed to give students long-term, realistic experiences with earning, spending, and saving money. The course is not taught from a textbook. Materials come from sources such as newspaper classified sections (i.e., jobs, apartments, used cars, and furniture), social security number applications, income tax forms, insurance forms and policies, and other everyday forms and charts. The instructor or curriculum developer should provide additional handouts and record sheets.

CONSUMER APPLIED MATHEMATICS

This course is similar to many consumer mathematics courses presently taught in Ohio high school. However, emphasis is shifted from personal computation skill development (or maintenance). Use of calculators and computers should be incorporated wherever feasible. Real world applications of mathematics are drawn from areas of student interest. A project, or hands-on, approach is recommended.

EXAMPLE OBJECTIVES (Note: this is one of many possible lists of objectives appropriate for a high school consumer mathematics course.)

The student will be able to:

- open and maintain a bank account.
- prepare a budget.
- apply mathematics to consumer decisions, including unit pricing and comparison shopping.
- judge advertising critically and detect deceptive or fraudulent practices.
- locate and use information about job opportunities for which he or she is qualified.
- fill out job application forms and present a resume when applying for a job.
- interpret his or her paycheck stub.
- make sound judgments regarding credit buying based on a mathematical understanding of the concepts and terms involved.
- determine credit cost.
- solve problems related to credit and borrowing.
- solve problems related to home ownership and renting of property (e.g., insurance, mortgages, deposits).

- solve problems related to utilities.
- solve problems related to travel by rail, bus, or plane.
- use travel reference sources.
- solve problems related to car ownership.
- prepare federal, state, and local tax returns.
- solve problems related to personal purchases.
- solve problems involving various taxes (i.e., gas, sales, property).
- solve problems related to insurance.

EXAMPLE ACTIVITIES (Consumer Simulation Mathematics)*

1. Entering the World of Work: Have students assume a hypothetical job. Students should *accept* a job advertised in the newspaper for which they actually possess the entry skills. Using realistic salary figures, have students prepare a budget.
2. Spending Money: Engage students in comparative shopping and unit pricing activities. Ask students to furnish a room or complete apartment. Mathematical topics such as measurement, geometry, ratio and proportion, fractions should be applied.
3. First Car: Help students explore car ownership. Students should work with down payments, monthly payments, gasoline mileage, insurance, and the costs of maintaining and repairing an automobile.
4. Going on Vacation: Have students read maps and brochures and calculate items such as distances, time, daily food costs, luggage weight and size, sharing costs, and budgeting for souvenirs.
5. First Apartment: Have students examine housing costs and options. Provide sample rental agreements and utility bills.
6. Buying Groceries: As students apply the concepts of comparative shopping and unit pricing, have them consider the nutritional aspects of planning meals. (Adapted from Lindsay, 1979, p. 76)

EXAMPLE ACTIVITIES (Consumer Applied Mathematics)

1. Newspaper and magazine articles provide the background for a variety of current problems. Using the following newspaper article and accompanying questions as examples, have students find newspaper or magazine articles (not advertisements), bring them to class, and generate as many mathematical questions or problems from the articles as they can. Then have students trade the questions or problems among themselves. All students must solve their own problems so that they can check the work of other students.

*Several computer programs of different types (simulation, drill and practice) which address these topics are available. If computers are available, the use of instructionally sound programs should be considered.

CONSUMER MATHEMATICS

PARK UNCHAINS BADDEST BEAST

KINGS MILLS, Ohio—The Beast is on the prowl.

The Beast is the new roller coaster at Kings Island Amusement Park. It was officially and literally unchained Friday.

Starting this weekend, the public will have the opportunity to ride the wooden monster that is being billed as "the biggest, baddest, longest, fastest coaster in the world."

During Friday morning ceremonies, three members of the U.S. Marine Corps snapped a link chain restraining the coaster's 40-seat car.

Officials of Taft Broadcasting Co., Kings Island's parent company, then took part in the maiden run of the $3.8 million roller coaster.

The experience lasted 3 minutes, 40 seconds.

The Beast is an appropriate name.

Covering 35 acres, The Beast has 7,400 feet of track, three vertical drops, three underground tunnels and a 628-foot helix—a large circle of track.

The first vertical drop has riders plummeting 135 feet at a 45-degree angle.

By the time The Beast zooms into a tunnel, it is traveling about 70 miles an hour.

Rain dampened Friday's dedication. Moments after the initial run, light rainfall turned to a downpour with thunder and lightning adding effects—and curtailing further runs of the coaster.

Kings Island wanted three things out of The Beast—thrills, speed and smoothness. The amusement center was also concerned about safety.

The Beast has a computerized safety system and closed circuit television, which monitors the entire track. There is also a height requirement—54 inches.

Persons under 4 feet 6 will not be permitted on the ride.

Kings Island opens for the season Saturday. It is open at 10 a.m. Saturdays and Sundays until May 20. Daily operation wil be from May 26 to September 3.

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ROLLERCOASTER UPS AND DOWNS

a. If coasters leave approximately every 30 seconds, how many trains are on the track at any given time?

b. Suppose each coaster carries 32 passengers. Approximately how many people ride in one day?

c. How far does one coaster travel in one day?

d. About how many people ride the rollercoaster during its regular season of daily operation?

e. About how far apart are individual coasters on the track?

f. About how many miles are traveled by coasters during a regular season?

(Meiring, 1980, p. 51)

2. Using the stock market quotations for one week, buy and sell the following stocks and figure your income and outgo for each day's trading. Compute each transaction on the basis of the last quoted price. Assume that you may act as your own broker so you do not need to worry about broker's fees. (Broker's fees are actually an important consideration, especially in short-term trading where price changes may be small.)

   Monday: Buy 200 shares of Revlon, 10 shares of General Motors, 150 shares of Coca-Cola, and 300 shares of IBM.

   Tuesday: Sell 100 shares of Revlon, 5 shares of General Motors, and 50 shares of IBM; buy 15 shares of Exxon and 35 shares of Texaco.

   Wednesday: Sell remaining shares of Revlon and General Motors; sell 50 shares of Coca-Cola; buy 30 shares of Exxon and 35 shares of Texaco.

   Thursday: Sell remaining shares of Coca-Cola and IBM; buy 25 shares of Exxon and 35 shares of Texaco.

   Friday: Sell all remaining shares of stock.

   Did you gain or lose money over the week's activity? How much?

EXAMPLE PROBLEMS (The following problems are intended to be illustrative of type, difficulty, and interest level.)

1. Carpeting is typically available in large rolls which come in the standard widths of 9 feet or 12 feet. You may buy any length you wish from such a roll even though it is customary to advertise the price in dollars per square yard. A room you wish to carpet will often be an approximate rectangle $l \times w$ feet. You may have to cut out a small piece because of a door or a radiator, but this will not affect what you buy.
   a. If you need to carpet a room whose dimensions are either 9 feet or 12 feet, the size roll from which to buy is apparent. But suppose the room is 7 feet × 10 feet. Which roll will be cheaper, assuming that each costs \$6 per square yard?
   b. Suppose the room is 7½ feet × 10 feet. Which roll is cheaper then?
   c. For what room sizes $l \times w$ is it cheaper to buy from the 9-foot roll? When do they cost the same?
   d. Suppose the narrower 9-foot width is more expensive; say \$8 per square yard. Is there any size room for which this is now cheaper? (MAA and NCTM, 1980, p. 186)

CONSUMER MATHEMATICS

2. A bank offers three types of checking accounts:
   - a system in which the service charge is based on the number of checks written and the minimum balance during the month according to the table below;
   - a system in which the service charge is 25¢ plus 10¢ per check;
   - a system in which there is a flat service charge of $2.00 unless the minimum balance is at least $200; then there is no service charge.

   a. Which system is best for an account with a minimum balance of $0 and 20 checks written monthly?

   b. Which system is best for an account with minimum balance of $250 and 20 checks written monthly?

   c. Suppose the $250 in part b could be withdrawn and deposited in a savings account at 5%. Would this be worth doing?

Account analysis chart—minimum balance during month

Number of Checks	Under $100.00	$100.00 $199.00	$200.00 $299.00	$300.00 $399.00	$400.00 $499.00	$500.00 $599.00	$600.0.. $699.00	$700.00 $799.00	$800.00 $899.00	$900.00 $999.00	Number of Checks
0	$ .75										0
1	.81	$ .66	$ .51	$ .36							1
2	.87	.72	.57	.42				No charge will be made to you if			2
3	.93	.78	.63	.48	$ .33			the number of your checks for the			3
4	.99	.84	.69	.54	.39			month falls within the blank area.			4
5	1.05	.90	.75	.60	.45	$ .30					5
6	1.11	.96	.81	.66	.51	.36					6
7	1.17	1.02	.87	.72	.57	.42					7
8	1.23	1.08	.93	.78	.63	.48	$ .33				8
9	1.29	1.14	.99	.84	.69	.54	.39				9
10	1.35	1.20	1.05	.90	.75	.60	.45	$ .30			10
11	1.41	1.26	1.11	.96	.81	.66	.51	.36			11
12	1.47	1.32	1.17	1.02	.87	.72	.57	.42			12
13	1.53	1.38	1.23	1.08	.93	.78	.63	.48	$ .33		13
14	1.59	1.44	1.29	1.14	.99	.84	.69	.54	.39		14
15	1.65	1.50	1.35	1.20	1.05	.90	.75	.60	.45	$ .30	15
16	1.71	1.56	1.41	1.26	1.11	.96	.81	.64	.51	.36	16
17	1.77	1.62	1.47	1.32	1.17	1.02	.87	.72	.57	.42	17
18	1.83	1.68	1.53	1.38	1.23	1.08	.93	.78	.63	.48	18
19	1.89	1.74	1.59	1.44	1.29	1.14	.99	.84	.69	.54	19
20	1.95	1.80	1.65	1.50	1.35	1.20	1.05	.90	.75	.60	20
21	2.01	1.86	1.71	1.56	1.41	1.26	1.11	.96	.81	.66	21
22	2.07	1.92	1.77	1.62	1.47	1.32	1.17	1.02	.87	.72	22
23	2.13	1.98	1.83	1.68	1.53	1.38	1.23	1.08	.93	.78	23
24	2.19	2.04	1.89	1.74	1.59	1.44	1.29	1.14	.99	.84	24
25	2.25	2.10	1.95	1.80	1.65	1.50	1.35	1.20	1.05	.90	25
26	2.31	2.16	2.01	1.86	1.71	1.56	1.41	1.26	1.11	.96	26
27	2.37	2.22	2.07	1.92	1.77	1.62	1.47	1.32	1.17	1.02	27
28	2.43	2.28	2.13	1.98	1.83	1.68	1.53	1.38	1.23	1.08	28
29	2.49	2.34	2.19	2.04	1.89	1.74	1.59	1.44	1.29	1.14	29
30	2.55	2.40	2.25	2.10	1.95	1.80	1.65	1.50	1.35	1.20	30
31	2.61	2.46	2.31	2.16	2.01	1.86	1.71	1.56	1.41	1.26	31
32	2.67	2.52	2.37	2.22	2.07	1.92	1.77	1.62	1.47	1.32	32
33	2.73	2.58	2.43	2.28	2.13	1.98	1.83	1.68	1.53	1.38	33
34	2.79	2.64	2.49	2.34	2.19	2.04	1.89	1.74	1.59	1.44	34
35	2.85	2.70	2.55	2.40	2.25	2.10	1.95	1.80	1.65	1.50	35
36	2.91	2.76	2.61	2.46	2.31	2.16	2.01	1.86	1.71	1.56	36
37	2.97	2.82	2.67	2.52	2.37	2.22	2.07	1.92	1.77	1.62	37
38	3.03	2.88	2.73	2.58	2.43	2.28	2.13	1.98	1.83	1.68	38
39	3.09	2.94	2.79	2.64	2.49	2.34	2.19	2.04	1.89	1.74	39
40	3.15	3.00	2.85	2.70	2.55	2.40	2.25	2.10	1.95	1.80	40
41	3.21	3.06	2.91	2.76	2.61	2.46	2.31	2.16	2.01	1.86	41
42	3.27	3.12	2.97	2.82	2.67	2.52	2.37	2.22	2.07	1.92	42
43	3.33	3.18	3.03	2.88	2.73	2.58	2.43	2.28	2.13	1.98	43
44	3.39	3.24	3.09	2.94	2.79	2.64	2.49	2.34	2.19	2.04	44
45	3.45	3.30	3.15	3.00	2.85	2.70	2.55	2.40	2.25	2.10	45
46	3.51	3.36	3.21	3.06	2.91	2.76	2.61	2.46	2.31	2.16	46
47	3.57	3.42	3.27	3.12	2.97	2.82	2.67	2.52	2.37	2.22	47
48	3.63	3.48	3.33	3.18	3.03	2.88	2.73	2.58	2.43	2.28	48
49	3.69	3.54	3.39	3.24	3.09	2.94	2.79	2.64	2.49	2.34	49
50	3.75	3.60	3.45	3.30	3.15	3.00	2.85	2.70	2.55	2.60	50

(MAA and NCTM, 1980, p. 64)

3. Mr. Briscoe has a job for which he is paid $5.20 an hour with time and a half for overtime (any time worked over forty hours in any one week). One week his time card indicated the following number of hours worked each day:

Monday:	8	Thursday:	10
Tuesday:	10	Friday:	8
Wednesday:	7	Saturday:	4½

   a. What was the gross pay due Mr. Briscoe for this week's work?

   b. Assume that the following deductions were taken from his paycheck for that week:

15%	Federal Income Tax
6%	State Income Tax
5¾%	Social Security Tax
3%	Pension Fund
$17.27	Medical Insurance
$ 8.00	Union Dues
$ 0.27	State Industrial Accident Insurance

   What will be the net amount of Mr. Briscoe's check?

   c. If Mr. Briscoe's employer had to equal the amount paid by Mr. Briscoe for his social security, his pension fund, and the medical insurance, and also had to pay $3.00 for the State Industrial Accident Insurance, what was the total cost to the employer of Mr. Briscoe's services that week?

(MAA and NCTM, 1980, p. 71)

PROBLEM SOLVING MATHEMATICS

This semester or year long course provides students with specific techniques and methods for solving mathematical problems. Problem solving should be the focus of much of school mathematics and should be incorporated into all courses to some degree. This course differs from most others because the primary objective is to teach the process of problem solving.

The course outline is adapted from a semester course developed in the Pittsburgh schools (Bower, 1983). A similar course could have been developed using materials distributed by the Ohio Department of Education (Meiring, 1980). A variety of textbooks and other references should be available: access to a computer is needed.

PROBLEM SOLVING MATHEMATICS

EXAMPLE OBJECTIVES (Note: this is one of many possible lists of objectives appropriate for a high school problem solving mathematics course.)

The student will be able to:

- exhibit self-confidence when attempting to solve problems.
- work as a member of a group toward a problem solution.
- read and restate a problem.
- list the given information provided and partial solutions that occur with the initial reading of a problem.
- identify the assumptions that are being made.
- tabulate specific examples that meet the conditions of a problem.
- draw a diagram illustrating the conditions of a problem.
- construct a physical model that represents the conditions of a problem.
- identify the different cases involved in a problem.
- develop patterns for solving problems involving numerical sequences.
- draw diagrams when counting is involved in problem solving.
- design a table using deductive reasoning.
- illustrate given facts using Venn diagrams.
- design a model of the conditions in a problem.
- solve problems in arithmetic, geometry, algebra, and logic.
- discover a reasonable approach to a problem, given the correct answer.
- analyze a detailed solution to a problem the student is unable to solve and transfer this knowledge to a related problem.
- develop techniques to take standardized tests.
- analyze a problem and identify the additional information needed.
- identify and apply materials needed for the design or construction of a given problem.
- develop critical thinking skills leading to creative modifications in the design of a project.
- develop a better appreciation of the importance of mathematics in the physical world.
- give examples in which mathematical skills are used by individuals in their daily lives.
- work in a consistent and persistent manner when solving mathematical problems and exercises.

- exhibit confidence in being able to apply personal skills and knowledge to attack and solve problems.
- work independently on problems or exercises for a reasonable time before seeking assistance. (Adapted from Bower, 1983, pp. 61-66)

EXAMPLE ACTIVITY

Classical problems, solved and unsolved, are a rich source for motivating students. Students should do independent research on at least one classical problem from mathematics. The research might include:

a. when the problem was first formulated.
b. whether or not the problem has been solved.
c. the evolution of the solution of the problem.

Problems or topics that might be considered include:

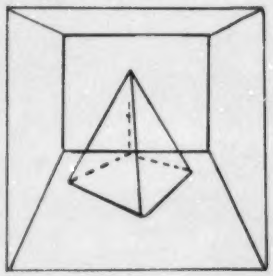
a. angle trisection.
b. squaring the circle.
c. Fermat's Theorem.
d. Goldbach's Conjecture.
e. Four Color Map.

Ideally, students should attempt an original solution or proof.

EXAMPLE PROBLEMS

1. In social studies, we studied the effects of two-children families versus three-children families in population education. Your task is to create a model of two-child family growth versus three-child family growth. Assume that two couples, age 20, without children, are placed upon the island of Fertilia. These couples and all their descendants are committed to having only two children per family. At the same time, two other couples, also age 20 and without children, are placed upon the neighboring island of Greater Fertilia. They and their descendants agree to have exactly three children per family. Show how the population will grow on these two islands over a period of 160 years. Make whatever assumptions you wish about when children are born, when people die, and so forth. But remember to account for deaths as well as births. (Meiring, 1980, p. 61).

VOCATIONAL MATHEMATICS



2. The problem task is to find the height of a pyramid from information which can be determined from the outside. Constructed in the classroom is a pyramid framework which you can enter. Take measurements, determine how the height can be found indirectly, and check your procedure by measuring. (Meiring, 1980, p. 59)
3. Your club will again handle concessions at the home football games this year. You need to place an order for food and drinks for the first game sales. Last season's total attendance for six home games was 13,593. Total food and drinks consumed were: 9,854 hot dogs, 6,512 hamburgers, and 21,678 soft drinks. This year's team is better so attendance is expected to be at least 15% higher. But price hikes for food are expected to lower demand by about 5%. How many hot dogs, hamburgers, and soft drinks should you order for the first game? (Meiring, 1980, p. 60)

EXAMPLE PROBLEMS

1. During one year a discount store made the following purchases of calculators:

Jan. 7	150 calculators @ $10	Aug. 3	50 calculators @ $ 8
May 27	150 calculators @ $12	Nov. 25	120 calculators @ $14

   On December 31, the date on which the inventory was taken, there were 130 calculators in the store. Because the store stocks all calculators together, it is impossible to obtain the specific purchase lots for the 130 calculators. Calculate the value of the 130 calculators in the inventory.
2. Party Mix, Inc., sells cashews for $4.75 a pound. Peanuts sell for $2.25 a pound. One popular party mix can be made by combining the cashews and the peanuts in a ratio of 3 to 7 respectively. For how much per pound should the new mixture sell to obtain the same profit as if the cashews and peanuts were sold separately?

VOCATIONAL MATHEMATICS

There are three options considered under vocational mathematics.* The first involves mathematics related to specific vocational areas. A common example is business mathematics. The second option is general applied mathematics. This course draws mathematical applications from a variety of vocational areas. It should provide an *overview* of mathematics from many career options. The third option is the *cluster approach* in which the mathematical skills necessary for different vocations are analyzed and grouped in such a way that students study necessary skills for entering various vocations. A brief course description is given below for both business mathematics and general applied mathematics, and the cluster approach is discussed.

*To qualify as a mathematics unit, all courses must be taught by a certificated mathematics teacher.

BUSINESS MATHEMATICS

BUSINESS MATHEMATICS

This alternative is designed for students intending to pursue a career in business but not desiring a four-year college program (for the latter, three years of college preparatory course work is recommended since calculus is now a requirement for many college business curricula). The course should utilize calculators and computers wherever feasible.

Business mathematics should focus on the application of algebra and arithmetic to common business situations. Depending on the length of the course, it may be possible to include additional topics such as concepts and skills related to probability and statistics, linear programming, and limits.

EXAMPLE OBJECTIVES (Note: this is one of many possible lists of objectives appropriate for a high school business mathematics course.)

The student will be able to:

- apply mathematics in a variety of retail merchandising situations.
- apply mathematics to markup procedures.
- apply mathematics in price determination and markdown.
- make determinations regarding inventory control and value.
- apply mathematics to discount and invoice procedures.
- use basic accounting procedures.
- determine payroll deductions.
- compute salary for various methods of remunerations.
- maintain records of employ payroll and deductions.
- compute commissions.
- compute discounts.
- determine depreciation.
- apply mathematics in data processing.
- reconcile a balance sheet.
- apply basic statistical procedures to business problems.
- read, interpret, and construct graphs.
- compute annuities.
- make business decisions based on the interpretation of mathematical information.

VOCATIONAL ORIENTATIONAL MATHEMATICS

EXAMPLE ACTIVITIES

1. Advertising is a rich source for classroom activities in statistics. Locate an advertisement that makes a statement of the type *three out of four people prefer product X.* Bring the advertised product and the advertisement to class. Divide the class into groups of four and determine what fractional part of each group uses the product. Discuss the results.
2. Using a computer software package such as VisiCalc, the class should keep the financial records for a school organization such as student council or pep club for the entire school year.

VOCATIONAL ORIENTATION MATHEMATICS

This course is intended for students who have specific vocationally oriented career goals. In addition to application and maintenance of arithmetic skills, the course should include algebraic, geometric, and trigonometric applications. Students should have access to calculators and computers whenever feasible.

EXAMPLE OBJECTIVES (Note: this is one of many possible lists of objectives appropriate for a high school vocational orientation mathematics course.)

The student will be able to:

- use a calculator for time-consuming calculations and to verify solutions.
- use *prepared* programs on a computer.
- use a computer to improve problem solving and mathematical skills.
- solve linear equations, systems of equations, and quadratic equations.
- apply algebraic methods to the solution of real world problems.
- name and use geometric line and angle relationships.
- apply geometric properties of triangles, quadralaterals, circles, parallelepipeds, cones, cylinders, and spheres to real world problems.
- name and use trigonometric ratios and properties.

EXAMPLE ACTIVITIES

1. Have the class construct cardboard models depicting three-dimensional geometric relationships such as the intersection of two planes or three planes. Have the class construct geometric solids such as cubes, tetrahedrons, and dodecahedrons.
2. Class members should design linoleum or tile patterns using principles and ideas associated with tesselation.

EXAMPLE PROBLEMS

1. A sculptor has a 16 foot steel bar which she wishes to cut and weld to make a cubical framework.
   a. If a bar is very thin, what is the largest cube the sculptor can make?
   b. If the bar has a 1.5″ square cross section so that there will be an overlap at the corners, what is the largest cube the sculptor can make? (MAA and NCTM, 1980, p. 84)
2. An automobile cooling system contains a mixture of water and antifreeze that freezes at 12°F. The owner wants protection against temperatures as low as -20°F. The antifreeze manufacturer publishes a table as follows:

% Antifreeze in mixture	Minimum safe temperatures
0	32°F
•	•
•	•
•	•
20	12
•	•
•	•
•	•
90	-20°F

   How much liquid should the mechanic drain from the cooling system and replace with pure antifreeze if the cooling system holds 8 liters of coolant? (MAA and NCTM, 1980, p. 85)
3. A contractor has agreed to lay a brick terrace in the form of a 12′ × 15′ rectangle. The bricks are to be laid against one another on a 3″ bed of sand. A standard brick measures 2¼″ × 3½″ × 8″. Sand weighs approximately 100 lbs. per cubic foot, and bricks weight about 2¼ ton per thousand.
   a. How many bricks are needed and how much do they weigh?
   b. How many cubic yards of sand will be needed? How much will the sand weigh?
   c. If the sand and bricks must be moved around a house in a wheelbarrow which is capable of carrying 60 lbs., how many wheelbarrow trips will be needed? (MAA and NCTM, 1980, p. 52)

VOCATIONAL CLUSTER MATHEMATICS

4. In determining the amount of fabric needed to make draperies, the following method is suggested by the Cooperative Extension Service of the University of Idaho. To determine length, measure from the top of the rod to the floor (or bottom of window apron, or sill) and add 3½" each for the top and bottom hems (7" total). To determine the width of the fabric for draw draperies, multiply the distance between brackets by 2½ and add 1½" for each side hem. A window is to be covered by a two piece drapery. The window is 31" from rod to sill, and the brackets are 63" apart. What size piece of material should be used? (MAA and NCTM, 1980, p. 31)

VOCATIONAL CLUSTER MATHEMATICS

This course is designed for students who will be advancing to a specific vocational program at the 11th or 12th grade. Highly flexible and skill oriented, the course combines practical, applied mathematics instruction with real-life tasks, career information, and on-the-job situations. Drawn from areas such as service industries, business, construction, distribution, auto mechanics, agriculture, and metal trades, a variety of career-oriented problems develops students' skills in solving the kinds of mathematical problems they will face in their future occupations. This experience promotes the development of problem solving skills through real-life situations. The course emphasizes student activities and uses of learning aids with carefully controlled reading levels. Student use of mechanical and electronic devices should be incorporated wherever feasible.

Before initiating this course, it is imperative that local districts analyze the necessary mathematical skills of all vocational programs in the district. This analysis will enable students to gain a solid understanding of the mathematical concepts necessary in their occupations. With this information, basic mathematical skills can be grouped for students entering varied occupational areas. In larger districts, separate courses can be established for the grouped concepts. In some districts with small enrollments, general core concepts, supplemented with individualized packets, can be designed for vocational students in the different skill areas.

One possible grouping of mathematical skills, adapted from the Great Oaks Joint Vocational School Mathematics Program, consists of basic skills linked with vocational areas:

VOCATIONAL CLUSTER MATHEMATICS

Twenty-eight basic skills areas: These areas should be reinforced through application problems. Application situations should include everyday living and vocational applications from all relevant vocational areas. We do not expect students to drill with numbers apart from real world contexts.

1. addition
2. subtraction
3. multiplication
4. division
5. reducing fractions
6. changing improper fractions to mixed numbers
7. equivalent fractions
8. converting to improper fractions
9. adding fractions
10. subtracting fractions
11. multiplying fractions
12. dividing fractions
13. reading decimals
14. changing fractions to decimals
15. changing decimals to fractions
16. changing fractions to percents
17. changing percents to fractions
18. changing decimals to percents
19. changing percents to decimals
20. rounding off decimals
21. adding decimals
22. subtracting decimals
23. multiplying decimals
24. dividing decimals
25. find the percent of a number
26. percents to find a number
27. percents of two given numbers
28. reading a linear scale

Consumer Skills: simple interest, compound interest, hourly wages, piece wages, commission income, paycheck deductions, income taxes, sales taxes, buying a car, buying a house, money skills.

Estimating: rapid approximate calculations by first rounding numbers; simple techniques for estimating quantity, length, distance, weight; deciding when a particular result is precise enough for the purpose at hand.

Measurement: measure distance, weight, time, capacity, and temperature; calculation of simple areas and volumes; use of both metric and customary measurement tools.

Basic geometry: area, circumference, perimeter, volume, angles (drawing and determining with a protractor); vocabulary such as perpendicular, parallel, horizontal, cubic, area.

Basic algebra: solving equations in one variable, operations with signed numbers, extracting square roots, scientific notation, exponents.

Basic trigonometry: solving right triangles, solving oblique triangles, Pythagorean theorem, interpolation.

VOCATIONAL CLUSTER MATHEMATICS

The grouping of mathematical skills makes it possible to cluster vocational areas by requisite mathematical skills.

CLUSTER I VOCATIONAL AREAS	CLUSTER II VOCATIONAL AREAS	CLUSTER III VOCATIONAL AREAS
auto collision	agricultural mechanics	cabinet making
banking and accounting	animal care	carpentry
broadcast arts	appliance repair	commercial electricity
building maintenance	auto mechanics	communication electronics
business office duplication	child learning	construction electricity
clerk typist	commercial art	drafting
clothing and fabrics	cosmetology	heating and air conditioning
commercial banking	diesel mechanics	industrial electricity
data processing	diversified health	industrial electronics
dental assistant	electro mechanical	law enforcement
executive secretary	farm management	machine trades
floor mechanic	firefighting	welding
homemaking	food merchandising	
interoffice education	graphics	
legal secretary	horsemanship	
multimedia	horticulture	
OWE	hotel/motel	
receptionist training	industrial maintenance	
retail merchandising	industrial mechanic	
sales and marketing	laboratory assistant	
senior office education	marine mechanic	
travel consultant	masonry	
word processing	meat processing	
	medical office assistant	
	park services	
	power mechanics	
	recreation fitness and health	
	restaurant operations	

Then appropriate mathematical skills can be assigned to each vocational cluster.

MATHEMATICAL SKILLS FOR CLUSTER I	MATHEMATICAL SKILLS FOR CLUSTER II	MATHEMATICAL SKILLS FOR CLUSTER III
Mathematics should include:	Mathematics should include:	Mathematics should include:
28 basic skills areas, consumer skills, and estimating.	All skills from Cluster I and measurement and basic geometry.	All skills from Cluster I and II and basic algebra and basic trigonometry.

EXAMPLE ACTIVITIES (Vocational Cluster Mathematics)

1. Students should choose one occupation from each of the three clusters and interview an individual in that vocation to determine the different applications of mathematics used by that individual. Some suggested questions:
   a. Do you use a calculator or computer?
   b. What would be a typical mathematics problem you might encounter on the job?

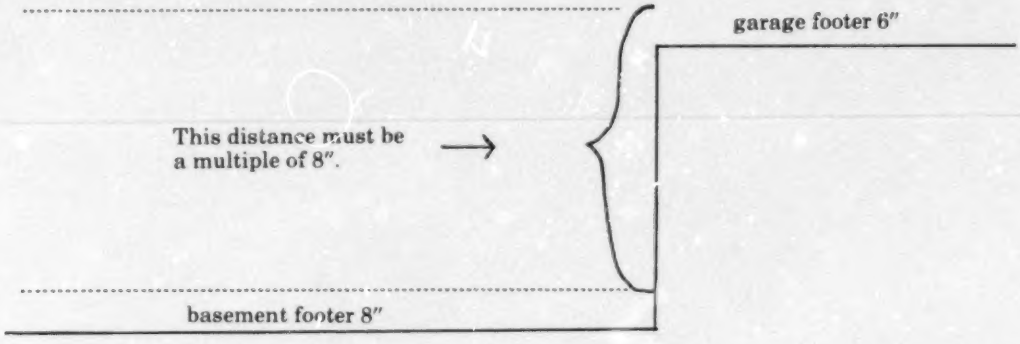
2. Ask local utility companies for instructions on reading meters. Students should read their home electric meters at daily intervals for two months. In addition to calculating average daily electric bills and monthly bills, the data should be graphed and used to compare electrical usage in homes of different sizes and ages.
3. Each year the Internal Revenue Service prepares a unit on computing income tax for high school students. Obtain this unit and prepare a tax return using hypothetical data concerning income and deductions.

EXAMPLE PROBLEMS

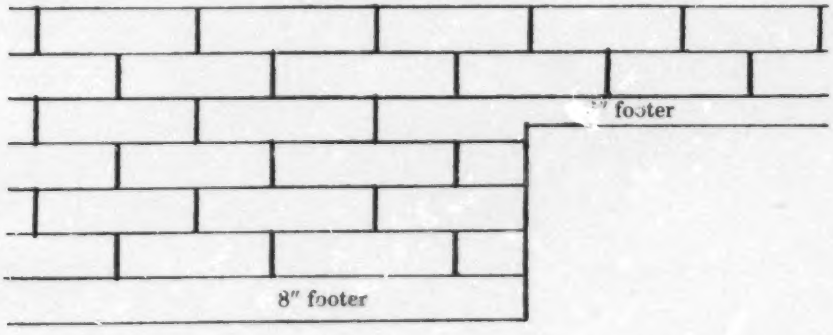
1. Two companies A and B, offer you a sales position. Both jobs are essentially the same, but A pays a straight 8% commission and B pays $51 per week plus 5% commission. The best sales people with either company rarely have sales greater than $4000 in any week. For what amounts of sales will company A pay more money? (MAA and NCTM, 1980, p. 82)
2. Wires carrying electricity are often placed in a protective steel pipe called *conduit*. (Some cities have building codes that require the use of conduit in certain types of structures.) A wiring job calls for 32 pieces of conduit 7½′ long, 3 pieces 7¾″ long, 8 pieces 13¼″ long and 6 pieces 9⅝″ long.
   a. Conduit comes in standard lengths of 10′. How many such lengths are needed for this job?
   b. How should the 10′ lengths in part (a) be cut to waste as little as possible? Since pieces less than 2′ in length are not normally carried from one job to another, such pieces are considered as waste. Try to cut the 10′ lengths so that there are as few pieces as possible with length less than 2′. (MAA and NCTM, 1980, p. 26)
3. The *feed efficiency* of any type of hog feed is defined as the average number of pounds of feed needed to produce 100 pounds of marketable pork. Typical feed efficiency values run from 250 to 550 pounds. Given current feed costs and the need for a balanced diet, a hog farmer decides to feed her hogs as follows:
   62% of feed A ($ 90 per ton, feed efficiency of 400 lbs.)
   24% of feed B ($110 per ton, feed efficiency of 420 lbs.)
   14% of feed C ($150 per ton, feed efficiency of 325 lbs.)
   a. What is the cost per ton of this feed mix?
   b. What is the feed efficiency of this feed mix?
   c. What is the average feed cost per 100 lbs. of marketable pork? (MAA and NCTM, 1980, p. 62)

4. The diameter of a circular ventilating duct is 140 mm. It is joined to a square duct system as pictured at right. To insure smooth air flow, the area of the circle and the square sections must be equal. What should be the length of a side of the square section?
(MAA and NCTM, 1980, p. 109)

5. *Figuring footer depth.** When digging at two different levels for a basement and a garage, the difference between transit readings must show a multiple of 8″ after making allowances for the footers. The basement footer will be 8″ deep. The garage footer will be 6″ deep. For example:



Blocks laid for the basement of the house must be evenly lined up with the blocks laid for the garage. Allowing for mortar between the blocks sets this measure at 8″ per row (course) of blocks. So, if the difference in measure between the footer in the basement and the footer in the garage isn't a multiple of 8″, the blocks will not fit correctly. See the illustration below:



*Prepared by Dianne Wengerd and Dan Blickensderfer, Coshocton Joint Vocational School, 1983.

Suppose the basement reading is 9'8" and the garage reading is 3'7". The basement footer will be 8" deep. That brings the depth back up 8" so we must subtract.

9'8" deep originally
- 8" back up for the footer to be poured
9' will be the depth after pouring the footer

The footer for the garage will bring that measurement back up 6". To find what that new measure will be we need to subtract the 6" from the depth we have now.

3'7" depth now
- 6" depth of footer to be poured
3'1" depth after footer poured

We now know the depths of the tops of the footers to be poured. Next we must convert the measures into inches.

9' = 108"
3'1" = 37"

Now we can find the distance between the tops of the two footers. Remember that this distance must be a multiple of 8" in order for the blocks to fit.

108"
- 37"
71"

71 is not a multiple of 8. What can you do to correct the situation?

Use the method described to figure the following measurements. The two transit readings are given. You are to figure out whether or not the block courses (rows) will fit correctly if the footers were poured and the blocks set according to those readings.

1. Basement reading 9'6"
   Garage reading 4'6"
2. Basement reading 10'
   Garage reading 4'8"
3. Basement reading 8'8"
   Garage reading 3'6"
4. Basement reading 9'4"
   Garage reading 5'
5. Basement reading 8'10"
   Garage reading 3'4"

MICROCOMPUTER APPLICATIONS

MICROCOMPUTER APPLICATIONS TO MATHEMATICS

This course will explore mathematical topics through use of the microcomputer. The emphasis will be on learning mathematics. Some time will be devoted to learning computer programming, however, so students can use the microcomputer to explore more fully the mathematics being studied.

The course assumes no prior experience with programming. If, in fact students enrolling in the course have significant facility with programming. The programming component can be minimized and more time spent on pure mathematical units. For students with little or no prior programming experience, at least six weeks should be spent on computer programming. Other lessons on programming may be necessary throughout the year as students determine that more sophisticated programming techniques or commands are necessary to solve specific mathematical problems. However, this is a mathematics course concerned primarily with ideas, concepts, skills, and problems of various branches of mathematics. The computer is a tool used to study mathematics; it is not the primary object of study in the course.

EXAMPLE OBJECTIVES (Note: this is one of many possible lists of objectives appropriate for a high school microcomputer applications course. In this sample, the programming language is BASIC. Some schools may prefer using PASCAL or some other structured language. A specific use language such as COBOL or FORTRAN is not recommended.)

The student will be able to:

- program a microcomputer using the statements PRINT, GO TO, INPUT, IF-THEN, DIM, FOR-NEXT, READ-DATA, RND, and GOSUB.
- use the computer to do computational tasks including evaluation of algebraic expressions.
- use the computer as a tool in doing basic geometric proof.
- use the computer to study geometric properties of lines and plane figures.
- use the graphic capabilities of the microcomputer to reproduce figures and create original graphic designs.
- simulate real-life and laboratory situations on the computer.
- use the computer to aid in the solution of a variety of algebraic, geometric, trigonometric, and other mathematical problems.
- create original games on the computer.

NOTE: The specific mathematical content of the course should be selected in terms of local program objectives and student needs. However, it is recommended that efforts be made to choose mathematical content that will make this course accessible to the largest student audience possible.

EXAMPLE ACTIVITIES

1. This activity develops a program to simulate a tontine. A tontine is a primitive (and now illegal) form of life insurance. In a tontine, each member of a group contributes a certain amount of money and agrees that the last survivor of the group will win the whole sum. (Can you guess why tontines are now illegal?) It is possible to develop a formula for the probability that a given participant will win the tontine, but the derivation is tedious and not very instructive. Therefore a simulation is appropriate. The simulation can be run a large number of times to obtain statistical estimates for such parameters as the probability that a particular person will win or the waiting time for a winner.

   To simulate the operation of a tontine, have students:

   a. Create a list of names and ages to represent the participants. Remove names from the list by a random process to simulate the gradual dying off of the group. When one name is left, the process stops, and that person is the winner.

   b. Obtain a table of mortality factors. Each mortality factor is the probability that a person will die within one year. Except for advanced ages, these probabilities will be rather small; for example, one estimate used by some insurance companies for dying within one year of age 60 is 0.013119. It is usually assumed that for some advanced age, such as 110, the probability will be 1, that is, death within one year at that age is almost certain.

   c. The death process might be set up in the following way. Take each name in turn, calculate that person's age; then use the mortality factor for that age (the probability of dying within one year) and a random number generator to simulate the occurrence or nonoccurrence of the event *dies within the year*. (Adapted from Kellogg, 1981, pp. 225-226)

2. Have students investigate area-perimeter relationships of rectangles. Students should write a program that will generate and print areas of figures when a constant perimeter is input. The length and width should be varied. For example, if the perimeter is to be 16 feet, then the figure might be 2 feet by 6 feet with an area of 12 square feet, or 4 feet by 4 feet with an area of 16 square feet. The student should identify limiting factors (i.e., whether or not dimensions should be integral). A goal of this activity is for students to make a generalization about the *shape* that will maximize the area enclosed.

COMPETENCY MATHEMATICS

3. The computer allows students to study topics requiring tedious calculations, such as compound interest. Using the formula $S = P(1 + i)^n$ (where P is the principal, i is the annual interest rate, and n is the number of interest periods), students could develop a computer program that would allow them to:
   a. compare outcomes if a fixed interest is compounded daily, monthly, quarterly, semiannually, and annually.
   b. determine the length of time it would take an investment to double at a given interest rate.
   c. determine the interest rate that would be necessary for an investment to double in a fixed time.

   The activity might have students verify their results with tables used by financial institutions.

EXAMPLE PROBLEMS: (Microcomputer Applications to Mathematics)

1. Given the coordinates for the vertices of a triangle, write a program to determine whether:
   a. the triangle exists.
   b. the triangle is equilateral.
   c. the triangle is right and isosceles.
   d. the triangle is isosceles but not equilateral.
   e. the triangle is right but not isosceles.
   f. the triangle is scalene.
2. Write a program which, for each pair of inputted numbers x and y, will compute and print the arithmetic mean $A = (x + y)/2$, the geometric mean $G = \sqrt{XY}$, and the harmonic mean $H = 2XY/X+Y$.
3. Write a program that will generate and print the primes less than 1000. Write a program to generate and print the first four perfect numbers. Print Pascal's triangle.
4. Write a program to produce magic squares.
5. Write a program to produce the first 100 Fibonnaci numbers. (Note: Rn 100th is about 20 digits, the 500th is about 100 digits.)

COMPETENCY MATHEMATICS

As part of the competency program, minimum standards require periodic assessment including the administration of a districtwide test at least once during grades nine through eleven. Competency mathematics assists students not meeting the district's competency requirements. Content for this course will vary from district to district. Class structure and activities should provide intervention instruction to remedy low performance in competencies identified by the periodic assessment or testing.

COMPETENCY MATHEMATICS

This course should use alternative methods of instruction. Because students have received instruction on these performance objectives but they did not achieve the desired learning outcomes, course instructional strategies should be varied. Individual or small group teaching, diagnostic-prescriptive approaches, and nongraded developmental materials are recommended components of this course.

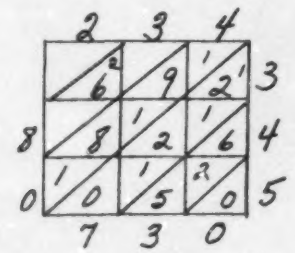
The competency mathematics course should be the only remedial mathematics course offered at the high school level. All other courses should consist of new content or new methods of applying content.

EXAMPLE ACTIVITIES

1. Some districts will have emphasized the mastery of computation as a part of their competency based education program in mathematics. The appropriateness of requiring students to master computational skills before going on to meaningful problem solving has long been questioned by mathematics educators. If, however, a school district feels such mastery is imperative and chooses to spend time on the development of computational skills at the secondary level, alternative methods of instruction and, in some cases, alternative algorithms should be used. Sources for developing meaningful computational activities include *Developing Computational Skills: The 1978 Yearbook of the National Council of Teachers of Mathematics,* and *Computation: A Changing Perspective* (Meiring, 1982).

   a. Have students use a lattice to do multidigit multiplication. This method is effective for those students who have difficulty in correctly lining up partial products.

```
   234
 × 345
 ----
  1170
  936
 702
 ----
 80730
```



COMPETENCY MATHEMATICS

b. Alternative algorithms can give some students success who are having difficulty with a standard algorithm.

Simplified borrowing technique:

"Borrowing" works rather effectively unless you need to borrow and are confronted with one or more zeros. Then you have to borrow more than once to carry out the original subtraction subproblem. This method uses place value to avoid multiple borrowing.

1. Align the numbers vertically and examine the digits in the one's column. Subtract the subtrahend digit from the minuend digit, if possible, and record.
2. If not possible, borrow one from the adjacent digit to the left in the minuend and increase the one's digit by ten. Carry out step one.
3. If when attempting to borrow, the ten's digit of the minuend is a zero, continue to the left in the minuend until a non-zero digit occurs. Treat all digits from the ten's zero to this non-zero digit as a separate number. "Borrow" one from this number, crossing it out and decreasing by one, and carry out step two.
4. Repeat steps 1 to 3 as necessary in working from right to left until the process is completed. (Meiring, 1982, p. 14)

```
 4003
-2857
```

```
 4003
-2857
```

```
 399
 ~~400~~3
-2857
```

```
 399
 ~~400~~3
-2857
 1146
```

c. Have students use a hundreds chart to find numbers that are ten greater than a given number. They should see, for example, that the number that is 10 greater than 15 is one row below it (see Figure 1). Then help them see that finding the number that is 10 greater than 15 is equivalent to finding the sum of 15 and 10. Give several practice exercises in which students (a) locate a number and (b) find the sum of that number and 10. (Example: Find 28. Now show the sum for 28 + 10.) When they are proficient at this exercise, move to finding the number that is 10 less than a given number. Repeat the approach, this time adding 20 and 30 to a number as well as subtracting 20 and 30 from the number.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40

Figure 1

COMPETENCY MATHEMATICS

One additional step is to have students solve examples such as 15 + 23 using the procedure shown in Figure 2. Both of these activities encourage students to add a number of tens to the whole first addend. (Trafton, 1978, p. 210)

Figure 2

1	2	3	4	5	6	7	8	9	10
11	12	13	14	(15)	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	~~36~~	~~37~~	~~38~~	39	40
				▼					

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2. Using newspaper advertisements provided by the teacher, make up problems to send to the fourth, fifth, and sixth grade mathematics classes. Be sure to provide answer sheets for the teacher. Below is an example for a sixth grade class.
   a. Mrs. Smith wants to take her children, Suzi (age 9), Johnny (age 7), and Mary (age 4) to the zoo. How much will the roundtrip transportation cost?
   b. If the trip takes 45 minutes, how long can the Smiths be at the zoo?
   c. Ali, Tal, Sammy, and Sarah are all 14 years old. If Sammy is going to pay the bus fare *to* the zoo for all four, how much money would he have to spend at the zoo if he started with $10.00?
   d. The Zoo Express Bus has a 50% roundtrip fare discount on July 4. How much will it cost the Dirty Dozen gang (all 11 years old) for transportation to and from the zoo on July 4?

COMPETENCY MATHEMATICS

EXAMPLE PROBLEMS

1. If you bought 12 cans of peaches that cost $.39 a can, how much would you pay for the peaches?
2. If you had 285 pieces of candy and wanted to put 15 pieces into a package, how many packages could you make?
3. If Marsha buys a pair of shoes that cost $10.98 and the sales tax is 4%, how much tax will she pay on the shoes?
4.

MENU

1.	Hamburger	$.60
2.	Cheeseburger	.65
3.	French fries	.25
4.	Milk shake	.55
5.	Apple pie	.35

Using the menu above, which combination of items can be bought with $1.00 and leave the least amount of change?

A. 1 and 5

B. 1 and 4

C. 2 and 4

D. 3, 4, and 5

5. How much does it cost to send a four-pound box to Zone 6?

Parcel Post Zone Rates

Weight—1 pound and not exceeding (pounds)	Zones							
	Local	1 and 2	3	4	5	6	7	8
2.....	$0.61	$0.72	$0.75	$0.83	$0.92	$1.03	$1.12	$1.21
3.....	.65	.78	.83	.92	1.04	1.18	1.32	1.45
4.....	.69	.84	.90	1.01	1.16	1.33	1.51	1.69
5.....	.73	.90	1.10	1.28	1.49	1.71	1.93	
6.....	.77	.96	1.05	1.19	1.40	1.64	1.90	2.17
7.....	.81	1.02	1.13	1.28	1.52	1.80	2.10	2.41
8.....	.85	1.08	1.20	1.37	1.64	1.95	2.29	2.65
9.....	.89	1.14	1.28	1.46	1.76	2.11	2.49	2.89
10.....	.93	1.20	1.36	1.55	1.88	2.26	2.68	3.13

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NOTES

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Total copies printed: 4,000
Unit cost: $.8430
Publication date: 1/84
(Includes paper costs)